Precalculus – Summer Packet

This summer packet is for students ENTERING PRECALCULUS AND HONORS PRECALCULUS.

Welcome to the world of Precalculus, there is a major shift in expectation for students to be able to recall many skills at a moment's notice. Certain concepts have been taught to you over the previous years are assumed to be mastered. If you do not have these skills, you will find that you will consistently get problems incorrect next year as you make mistakes. It is frustrating for students who spend much of their homework time relearning algebra concepts in addition to learning how to tie the concepts together. This summer packet is intended for you to brush up and possible relearn these topics.

On the following pages, you have assorted problems related to specific topics. Each problem should be done in the space provided. Rather than give you a textbook to remind you of the formulas and techniques necessary to solve the problem, there are a few websites that will be listed that have full instructions and brief refreshers at the front of some topics. Don't fake your way through these problems. You are only setting yourself up for a future struggle!

Realize also that many concepts are interrelated. This will be the focus throughout next year as we examine the mathematical relationships between topics numerically, algebraically, and graphically. While you may be strong in one of these approaches, you must learn to view each topic from the other two approaches as well to achieve full understanding. Success on your tests and quizzes throughout the year will depend on you being able to do so.

We want you to get off to a good start so spend some quality time on this packet this summer. Work needs to be shown when needed. Also, do not rely solely on the calculator to work through but you are expected to have strong calculator skills.

This packet is to be completed by the first day back to school August 9th, 2017. This summer packet is a study guide for your first assessment in the course. The packet itself will not be graded; instead you will be assessed over the content a few days after returning to school. You will work through any questions you have with your teachers in the first couple of days so come prepared to lead discussion! You will also be expected to efficiently work through the problems under a time constraint. Many students are not prepared for this expectation and find they do not have the time to check their answers like they are used to. Prepare accordingly.

It is a mistake to do this now. Let it go until mid-summer. We want these techniques to be relatively fresh in your mind in the fall. But, do not wait to do them at the very last minute. These take time. If you do a few concepts a day, the whole packet will take you about a week to complete.

We hope you take this seriously as we sincerely wish for you to be successful throughout this next year. Your preparation over the summer will be rewarded in unexpected ways during the year.

Good Luck!!

Use www.khanacademy.org to find specific math related topics with accompanying videos.

Use www.patrickjmt.com to find specific math related topics with accompanying videos.

Use www.youtube.com to find specific math related topics with accompanying videos.

Radicals:

Radicals:

To simplify means that 1) no radicand has a perfect square factor and
2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find a perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply both the numerator and the denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –
multiply the numerator and the denominator by the ***conjugate*** of the denominator
The ***conjugate*** of $3+\sqrt{2}$ is $3-\sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$
5. $\sqrt{\frac{11}{9}}$
6. $\left(\sqrt{5}-\sqrt{6}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Rationalize.

7. $\frac{1}{\sqrt{2}}$
8. $\frac{3}{2-\sqrt{5}}$

Complex Numbers:

Form of complex number - $a+bi$

Where a is the "real" is part and bi is the "imaginary" part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

- To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\cdot\sqrt{5}$ Pull out $\sqrt{-1}$

$= i\sqrt{5}$ Make substitution

Example: $(i\sqrt{5})^2 = i\sqrt{5}\cdot i\sqrt{5}$ List twice

$= i^2\sqrt{25}$ Simplify

$= (-1)(5) = -5$ Substitute

- Treat i like any other variable when +, -, ×, or ÷ (but always simplify $i^2=-1$)

Example: $2i(3+i) = 2(3i)+2i(i)$ Distribute

$= 6i+2i^2$ Simplify

$= 6i+2(-1)$ Make substitution

$= -2+6i$ Simplify and rewrite in complex form

- Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator **RATIONALIZE!!**

Simplify.

9. $\sqrt{-49}$

10. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

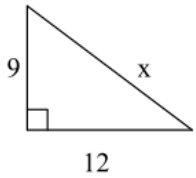
14. $\dfrac{1+6i}{5i}$

Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

15.



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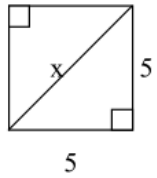

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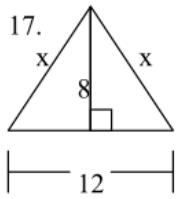
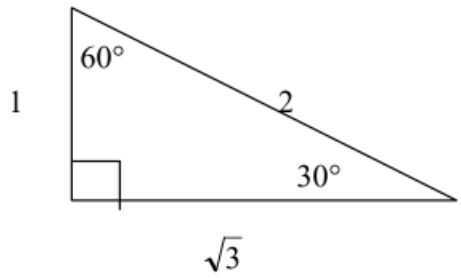
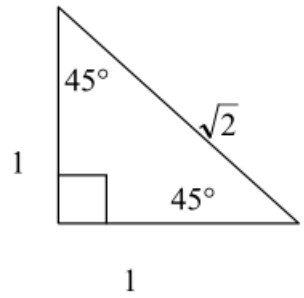


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30° - 60° - 90°$ triangles, sides are in proportion $1, \sqrt{3}, 2$.

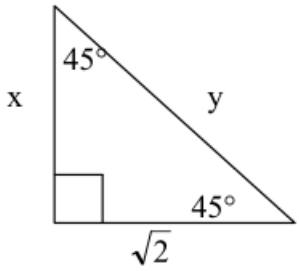


*In $45° - 45° - 90°$ triangles, sides are in proportion $1, 1, \sqrt{2}$.



Solve for x and y.

19.



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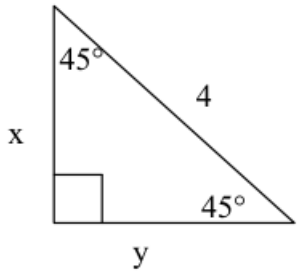

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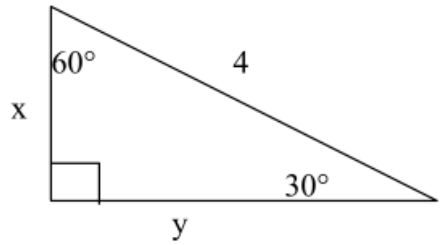

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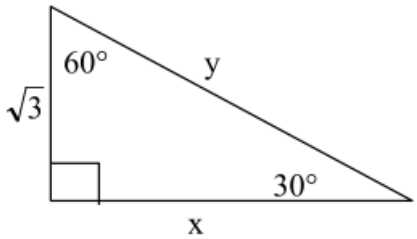


Equations of Lines:

Slope intercept form: $y = mx + b$	**Vertical line:** x = c (slope is undefined)
Point-slope form: $y - y_1 = m(x - x_1)$	**Horizontal line:** y = c (slope is 0)
Standard Form: $Ax + By = C$	**Slope:** $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: 5x – 4y = 8.

24. Find the x-intercept and y-intercept of the equation: 2x – y = 5

25. Write the equation in standard form: y = 7x – 5

Write the equation of the line in slope-intercept form with the following conditions:

26. slope = -5 and passes through the point (-3, -8)

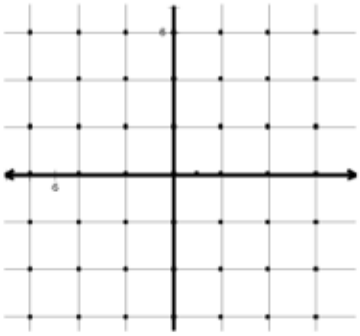
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

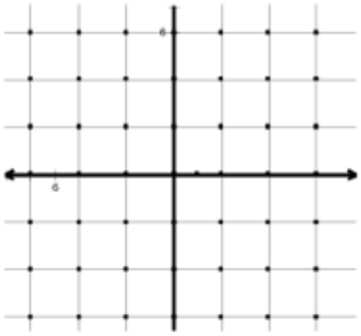
<u>Graphing:</u>

Graph each function, inequality, and / or system.

29. $3x - 4y = 12$

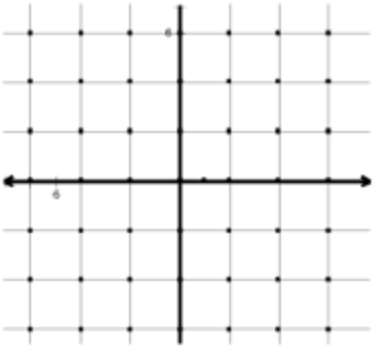


30. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$

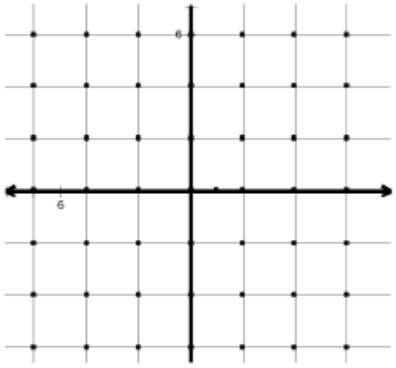


31. $y < -4x - 2$

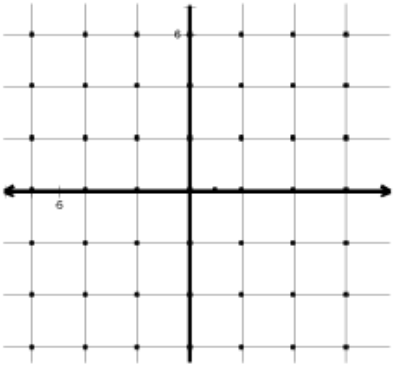
32. $y + 2 = |x + 1|$



33. $y > |x| - 1$



34. $y + 4 = (x - 1)^2$



Vertex: _____

x-intercept(s): _____

y-intercept(s): _____

Systems of Equations:

$3x + y = 6$

$2x - 2y = 4$

Substitution:
Solve 1 equation for 1 variable.
Rearrange.
Plug into 2nd equation.
Solve for the other variable.

Elimination:
Find opposite coefficients for 1 variable.
Multiply equation(s) by constant(s).
Add equations together (lose 1 variable).
Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	solve 1st equation for y	$6x + 2y = 12$	multiply 1st equation by 2
$2x - 2(6 - 3x) = 4$	plug into 2nd equation	$2x - 2y = 4$	coefficients of y are opposite
$2x - 12 + 6x = 4$	distribute	$8x = 16$	add
$8x = 16$	simplify	$x = 2$	simplify
$x = 2$			

Plug $x = 2$ back into original

$3(2) + y = 6$

$6 + y = 6$

$y = 0$

Solve each system of equations. Use any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\dfrac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\dfrac{3c}{c^{-1}}$

40. $\dfrac{2ef^{-1}}{e^{-1}}$

41. $\dfrac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $(a^3)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add / subtract polynomials, combine like terms.

EX: $8x - 3y + 6 - (6y + 4x - 9)$ *Distribute the negative through the parantheses.*

$= 8x - 3y + 6 - 6y - 4x + 9$ *Combine terms with similar variables.*

$= 8x - 4x - 3y - 6y + 6 + 9$

$= 4x - 9y + 15$

Simplify.

46. $3x^3 + 9 + 7x^2 - x^3$

47. $7m - 6 - (2m + 5)$

To multiplying two binomials, use FOIL.

EX: $(3x - 2)(x + 4)$ *Multiply the first, outer, inner, then last terms.*

$= 3x^2 + 12x - 2x - 8$ *Combine like terms.*

$= 3x^2 + 10x - 8$

Multiply.

48. $(3a + 1)(a - 2)$

49. $(s + 3)(s - 3)$

50. $(c - 5)^2$

51. $(5x + 7y)(5x - 7y)$

Factoring.

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a.) If you have one (other than 1) factor it out front.
b.) If you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a.) Is it difference of two squares? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b.) Is it sum or difference of two cubes? $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

EX: $m^3 + 64 = (m+4)(m^2 - 4m + 16)$
$p^3 - 125 = (p-5)(p^2 + 5p + 25)$

3 Terms

$x^2 + bx + c = (x+\quad)(x+\quad)$ **Ex:** $x^2 + 7x + 12 = (x+3)(x+4)$

$x^2 - bx + c = (x-\quad)(x-\quad)$ $x^2 - 5x + 4 = (x-1)(x-4)$

$x^2 + bx - c = (x-\quad)(x+\quad)$ $x^2 + 6x - 16 = (x-2)(x+8)$

$x^2 - bx - c = (x-\quad)(x+\quad)$ $x^2 - 2x - 24 = (x-6)(x+4)$

4 Terms -- Factor by Grouping

a.) Pair up first two terms and last two terms
b.) Factor out GCF of each pair of numbers.
c.) Factor out front the parentheses that the terms have in common.
d.) Put leftover terms in parentheses.

Ex: $x^3 + 3x^2 + 9x + 27 = \left(x^3 + 3x^2\right) + \left(9x + 27\right)$
$= x^2\left(x+3\right) + 9\left(x+3\right)$
$= \left(x+3\right)\left(x^2+9\right)$

Factor completely.

52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4 - 15b^2$

56. $9c^2 + 30c + 25$

57. $9n^2 - 4$

58. $27z^3 - 8$

59. $2mn - 2mt + 2sn - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use quadratic formula.

EX: $x^2 - 4x = 21$ *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$ *Now factor.*

$(x+3)(x-7) = 0$ *Set each factor equal to zero.*

$x + 3 = 0 \quad x - 7 = 0$ *Solve each for x.*

$x = -3 \quad x = 7$

Solve each equation.

60. $x^2 - 4x - 12 = 0$

61. $x^2 + 25 = 10x$

62. $x^2 - 14x + 40 = 0$

Rational Algebraic Expressions:

Multiplying and Dividing.
Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

EX:

$$\frac{x^2+10x+21}{5-4x-x^2} \bullet \frac{x^2+2x-15}{x^3+4x^2-21x}$$ *Factor everything completely.*

$$=\frac{(x+7)(x+3)}{(5+x)(1-x)} \bullet \frac{(x+5)(x-3)}{x(x-3)(x+7)}$$ *Cancel out common factors* in *the top and bottom.*

$$=\frac{(x+3)}{x(1-x)}$$ *Simplify.*

Simplify.

63. $\frac{5z^3+z^2-z}{3z}$

64. $\frac{m^2-25}{m^2+5m}$

65. $\frac{10r^5}{21s^2} \bullet \frac{3s}{5r^3}$

66. $\frac{a^2-5a+6}{a+4} \bullet \frac{3a+12}{a-2}$

67. $\frac{6d-9}{5d+1} \div \frac{6-13d+6d^2}{15d^2-7d-2}$

68. Function Names

Match the following equations to their description.

_____1.	$f(x)=\frac{2}{3}\lvert 4x+5\rvert-3$	A.	Linear Function
_____2.	$f(x)=\frac{2}{3}\sqrt[3]{4x+5}-3$	B.	Quadratic Function
_____3.	$f(x)=\frac{2}{3}\cdot\frac{1}{4x+5}-3$	C.	Absolute Value Function
_____4.	$f(x)=\frac{2}{3}(4x+5)^4-3(4x+5)^2-2$	D.	Cubic Function
_____5.	$f(x)=\frac{2}{3}(4x+5)^3-3$	E.	Cube Root Function
_____6.	$f(x)=\frac{2}{3}(4x+5)-3$	F.	Square Root Function
_____7.	$f(x)=\frac{2}{3}(4x+5)^2-3$	G.	Rational Function
_____8.	$f(x)=\frac{2}{3}\sqrt{4x+5}-3$	H.	Polynomial Function