

# **Entering**

# AP Calculus AB

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# AP Calculus BC

# Summer Math Packet

Summer 2020

Dear Calculus Students.

As we have completed a unique semester we have decided summer packets are more important than ever, given we had to do half of the spring semester remotely.

Therefore, in order to be successful in AP Calculus, you must be proficient at solving and simplifying each type of problem in this packet. This is a review of Algebra and Pre-Calculus. Upon returning to school in August, you are expected to have completed this assignment. Please make sure to show all work for each problem. I will answer questions pertaining to this packet during the first week of school and having your work to look back on will be beneficial to you in understanding the topic.

Graphing calculators may be used but I encourage you to be sure to know how to work problems without using a calculator as well. Half of the AP Calculus test prohibits any form of calculator use. It is highly recommended that you have either a TI-84 or TI-89 graphing calculator when the school year begins. FCS will have TI-89 graphing calculators available for check-out.

AP Calculus is a fast-paced course that is taught at the college level. Therefore, you must maintain all pre-requisite skills. We will not have time to spend re-learning the content in this packet. Please make sure you are comfortable with this material.

Here are some helpful websites to use, if needed:

- www.khanacademy.org
- www.patrickjmt.com
- www.youtube.com to find specific math related topics with accompanying videos

Enjoy!!

Mrs. Culbreth

# **SLOPE AND EQUATION OF LINES**

Write an equation of the line described in both slope intercept form and point slope form.

- 1. The line through (1, 4) and (3, 6)
- 2. The line through (5, -2) and (-5, 4)

- 3. The line through (2,1) with slope 4
- 4. The line with slope  $\frac{7}{2}$  and passing through (-2, -5)

The line with slope 8 and y-intercept 9

6. The line with slope  $-\frac{3}{8}$  passing through (0, 5)

- 7. The line through (-2, -8) and parallel to the line y = 5x 3
- 8. The line perpendicular to  $y = \frac{4}{5}x-9$  and passing through (8, -13)

## **SYSTEMS OF EQUATIONS**

#### **GRAPHING:**

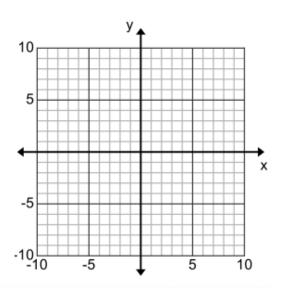
Solve each system by graphing. Make sure each equation is in slope intercept form (y = mx + b)!

1. 
$$2x + y = 2$$
  
 $y = x - 4$ 

The solution to the system of equations is

\_\_\_\_\_

Check your answer!



### **SUBSTITUTION:**

Solve each system by substitution. Check your answer.

2. 
$$4x + 7y = 19$$
  
 $y = x + 9$ 

$$3. \quad 4x - y = 2$$
$$2x + y = 10$$

#### **ELIMINATION:**

Solve each system of equations by elimination. Check your solution.

$$4. \quad 3x + 2y = 2$$
$$x - 2y = 6$$

$$5. \quad x + y = -1$$
$$2x + 3y = 0$$

#### **FACTORING**

GCF:

1. 
$$30x^5y^3 + 18x^3y^4 + 6x^3y^2$$

2. 
$$2x^2y - 4y^2z + 8xz^3$$

Trinomials (a = 1):

3. 
$$x^2 + 15x + 26$$

4. 
$$2x^4 + 18x^2 - 20$$

Trinomials  $(a \neq 1)$ :

5. 
$$5x^2 - 18x + 9$$

6. 
$$16x^2 + 60x - 100$$

Difference of Squares:

7. 
$$8x^2 - 162$$

8. 
$$x^4 - 64$$

Sum/Difference of Cubes:

9. 
$$8x^3 - 125y^6$$

10. 
$$81x^3 + 3y^3$$

Perfect Square Trinomials:

11. 
$$9x^2 + 24x + 16$$

12. 
$$x^2 - 8x + 16$$

Grouping:

13. 
$$5x^3 + 9x^2 - 15x - 27$$

14. 
$$6x^3 + 8x^2 - 15x - 20$$

Quadratic Form:

15. 
$$x^4 - 6x^2 + 8$$

16. 
$$x^6 + 18x^3 + 17$$

# **SOLVING EQUATIONS**

Solve for x.

1. 
$$4(x+3)-3=2(4-3x)-4$$

5. 
$$4t^3 - 12t^2 + 8t - 24 = 0$$

2. 
$$15 + x - 2x^2 = 0$$

6. 
$$\frac{4}{x-3} - \frac{4}{x} = 1$$

3. 
$$5x^4 - 12x^3 = 0$$

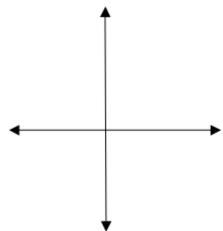
7. 
$$\sqrt{x-2} - 8 = 0$$

4. 
$$\frac{1}{x-2} = 3$$

8. 
$$(x+2)^{3/4} = 27$$

#### **FUNCTIONS**

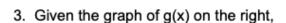
- 1. Given f(x) = -|x+3| 2
  - a. Sketch f(x)
  - b. Domain: \_\_\_\_\_
  - c. Range: \_\_\_\_
  - d. f(3) = \_\_\_\_
  - e. f(x+5) =
  - f. If f(x) = -3 then x = \_\_\_\_



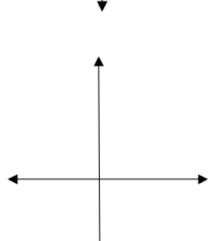
2. a. Graph the piece-wise function:

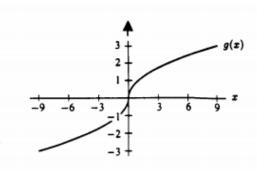
$$g(x) = \begin{cases} \frac{x}{2} & \text{if } x \ge 4\\ \sqrt{x} & \text{if } 0 < x < 4\\ x^2 & \text{if } x < 0 \end{cases}$$

- b. g(-3) = \_\_\_\_
- c. g(1) = \_\_\_\_
- d. g(0) = \_\_\_\_
- e. Is g(x) a continuous function? How do you know?

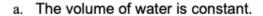


a. Estimate g(6) - g(0) = 6 - 0

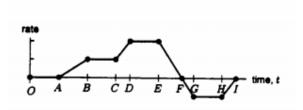




 The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch this line segment. 4. The rate at which water is entering a tank ( t > 0) is represented by the given graph. A negative rate means that water is leaving the tank. State the interval(s) on which each of the following holds true:



- The volume of water is decreasing.
- c. The volume of water is increasing.
- d. The volume of water is increasing fastest.



- 5. Given  $Q(x) = \frac{3x}{x+1}$ :
- a. Where is this function discontinuous?
- State the equation of the vertical asymptote
- State the equation of the horizontal asymptote

- d. Sketch the graph.
- e. Write the equation of the inverse of Q(x). (Switch the x & y and then rewrite as y = )
- 6. Use these functions to evaluate:

$$f(x) = x+1$$
  $g(x) = x^2 + 2x-3$   $h(x) = 2x-5$ 

$$h(x) = 2x - 5$$

a. 
$$f(3) =$$

b. 
$$g(5) =$$

c. 
$$h(\frac{1}{2}) =$$

d. 
$$g(-2) =$$

e. 
$$f(x) + h(x) =$$

f. 
$$g(x) - h(x) =$$

g. 
$$f(h(1)) =$$

h. 
$$h(g(-3)) =$$

i. 
$$h(f(x)) =$$

j. 
$$g(f(x)) =$$

### **EXPONENTS**

SIMPLIFY COMPLETELY:

1. 
$$2(x^4y^3)^0$$

$$2. \qquad \frac{15x^2}{5\sqrt{x}}$$

$$3. \qquad \frac{3c^2d^3}{\left(3cd^{-2}\right)^2}$$

$$4. \qquad \frac{\frac{2}{x^2}}{\frac{10}{x^5}}$$

$$6. \qquad \sqrt{x} * \sqrt[3]{x} * \sqrt[4]{x}$$

$$7. \ \frac{x^2-x+7}{x}$$

$$8. \ \frac{x^3 - x + 1}{\sqrt{x}}$$

$$9. \ \frac{\frac{1}{x+h} - \frac{1}{x}}{3h}$$

10. 
$$\frac{\frac{a}{a+1} + \frac{1}{a}}{\frac{1}{a} + \frac{1}{a+1}}$$

12. 
$$\ln e^{7}$$

14. 
$$e^0$$

15. 
$$e^{\ln x}$$

### **LOGARITHMS**

Solve for x.

1. 
$$\log_2 x = 3$$

2. 
$$\log_{\frac{1}{2}} x = 3$$

3. 
$$\log_3 81 = x$$

4. 
$$\log_3(-9) = x$$

5. 
$$\log_x 16 = -4$$

6. 
$$\log_x(\frac{1}{25}) = \frac{1}{2}$$

7. 
$$2^x = 3$$

8. 
$$2.43 \cdot 10^x = 1.84$$

9. 
$$\ln(x+5) = -\ln(x-1) - \ln(x+1)$$

10. 
$$3^{x+4} = 101$$

11. 
$$4e^{x+2} = 32$$

12. 
$$1.1 + \ln x^2 = 6$$

#### **UNIT CIRCLE**

For #1-2, find all 6 trig functions for:

1.) 
$$\frac{7\pi}{4}$$

$\sin\frac{7\pi}{4} =$	$\csc \frac{7\pi}{4} =$
$\cos \frac{7\pi}{4} =$	$\sec \frac{7\pi}{4} =$
$\tan \frac{7\pi}{4} =$	$\cot \frac{7\pi}{4} =$

2.) 
$$\frac{4\pi}{3}$$

$\sin\frac{4\pi}{3} =$	$\csc \frac{4\pi}{3} =$
$\cos \frac{4\pi}{3} =$	$\sec \frac{4\pi}{3} =$
$\tan\frac{4\pi}{3} =$	$\cot \frac{4\pi}{3} =$

- 3.) Find  $\theta$  if  $\cos \theta = \frac{1}{2}$  and  $\theta$  lies in Quadrant IV.
- 4.) Re-write the following in degrees:

a.) 
$$\frac{11\pi}{6}$$

b.) 
$$\frac{2\pi}{3}$$

- 5.) Evaluate:  $\sin \frac{-17\pi}{6}$
- 6.) What is the reference angle for 240°? Put your answer in degrees and in radians.
- 7.) Evaluate sin, cos, and tan  $-150^{\circ}$  without a calculator.
- 8.) Evaluate  $\csc \frac{-3\pi}{2}$
- 9.) Evaluate  $\cot \frac{-\pi}{2}$
- 10.) a.) List two angles coterminal to  $-120^{\circ}$ .
  - b.) Convert all three degree measures above to radians.
  - c.) Determine the sine, cosine, secant, cosecant, tangent, and cotangent of all three angles above.

#### **GRAPHING**

On graph paper, sketch a graph of each of the following functions. Identify and label any important points or features of the graph.

1. 
$$y = 3^x$$

$$2. y = \ln x$$

3. 
$$y = \log_4(x-5)$$

$$4. y = \sqrt[3]{x}$$

5. 
$$y = \sqrt[3]{x+2} - 4$$

$$6. x = y^2$$

7. 
$$y = \frac{3x-3}{(x^2-16)(x-1)}$$

8. 
$$y = x^4 - 4x^3 - 3x^2 + 6x + 2$$