AP CALCULUS AB/BC
SUMMER PACKET 2018

Dear Calculus Student:

In order to be successful in AP Calculus, you must be proficient at solving and simplifying each type of problem in this packet. This is a review of Algebra and Pre-Calculus. Upon returning to school in August, you are expected to have completed this assignment. We will correct and review during the first week of class, and then you will take a test on this content during the second week of class. Please make sure to show all work for each problem to receive full credit.

Calculators are NOT ALLOWED on this assignment except for the problems indicating they are permitted. 2/3 of the AP test is non-calculator. You must be proficient working problems without the use of a calculator. Additionally, 1/3 of the AP test requires the use of a graphing calculator. It is highly recommended that you have either a TI84 or TI89 graphing calculator when the school year begins. FCS will have TI89 graphing calculators available for check out.

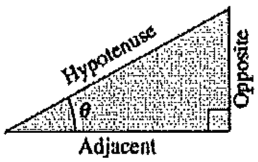
AP Calculus is a fast-paced course that is taught at the college level. Therefore you must maintain all pre-requisite skills. We will not have time to spend re-learning the content in this packet. Please make sure you are comfortable with this material.

Enjoy!!!

G.Culbreth

Formulas that must be memorized:

Right triangle definitions, where $0 < \theta < \pi/2$.

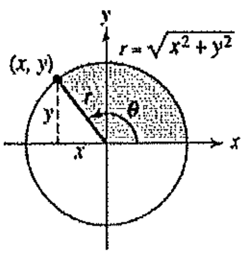


$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$

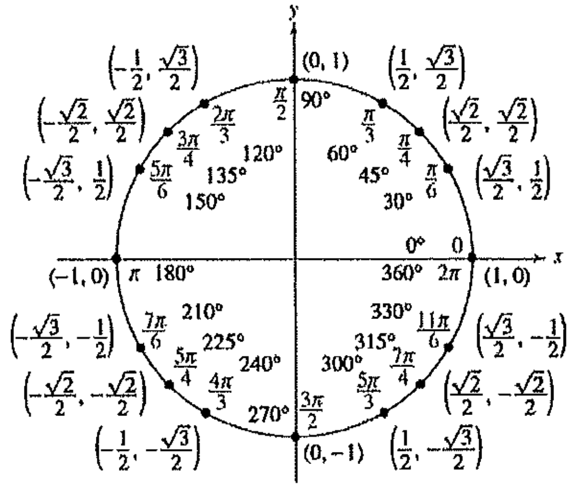
Circular function definitions, where θ is any angle.



$\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$



Reciprocal Identities

$\sin x = \dfrac{1}{\csc x}$ $\sec x = \dfrac{1}{\cos x}$ $\tan x = \dfrac{1}{\cot x}$

$\csc x = \dfrac{1}{\sin x}$ $\cos x = \dfrac{1}{\sec x}$ $\cot x = \dfrac{1}{\tan x}$

Tangent and Cotangent Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double-Angle Formulas

$\sin 2u = 2\sin u\cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

Geometric Formulas

Triangle	$A = \frac{1}{2}bh$
Equilateral Triangle	$A = \frac{\sqrt{3}}{4}s^2$
Circle	$A = \pi r^2$, $C = 2\pi r$
Sphere	$V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$
Cylinder	$V = \pi r^2 h$
Cone	$V = \frac{\pi}{3} r^2 h$

Exponents

$a^0 = 1,\ a \neq 0$

$a^1 = a$

$a^m \cdot a^n = a^{m+n}$

$\dfrac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{-m} = \dfrac{1}{a^m},\ a \neq 0$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Logarithms

$\ln 1 = 0$

$\ln e = 1$

$\ln mn = \ln m + \ln n$

$\ln\dfrac{m}{n} = \ln m - \ln n$

$\ln m^n = n\ln m$

$e^{\ln x} = x = \ln e^x$

$\log_b x = \dfrac{\ln x}{\ln a}$

Conversion formula:

$\log_b x = y$

$\Leftrightarrow$

$b^y = x$

Equations of lines

Slope-Intercept form $y = mx + b$

Point-Slope form $y - y_1 = m(x - x_1)$

Normal line is perpendicular to tangent line

Radicals

If $x^2 = a$, then $x = \pm\sqrt{a}$

Simplify each expression.

1. $\frac{2x^2+7x-4}{5x^2+20x}$

2. $\frac{6-\frac{5}{k}}{1+\frac{5}{k}}$

3. $\frac{\frac{a}{a+1}+\frac{1}{a}}{\frac{1}{a}+\frac{1}{a+1}}$

4. $\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

Factor each expression.

5. $(3x-4)^2+(x-5)(2)(3x-4)(3)$
6. $(5-2x)(3)(7x-8)^2(7)+(7x-8)^3(-2)$
7. $5(2x+1)^2+(5x-6)(2)(2x+1)(2)$

For each function, find the vertical asymptotes, horizontal asymptotes, and the x- and y- coordinates of any holes.

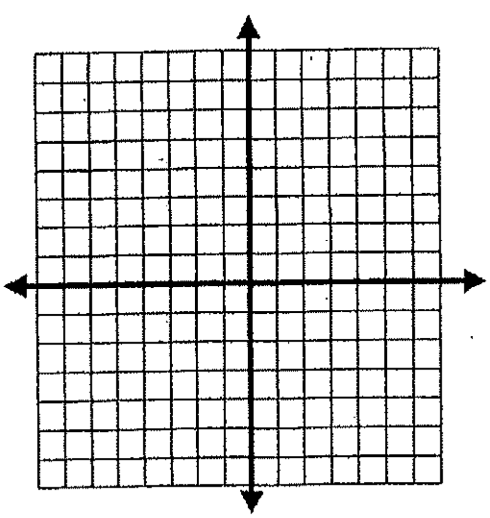
8. $f(x)=\frac{x^2-x-6}{x^2-4}$

9. $f(x)=\frac{4x^2-9}{2x+3}$

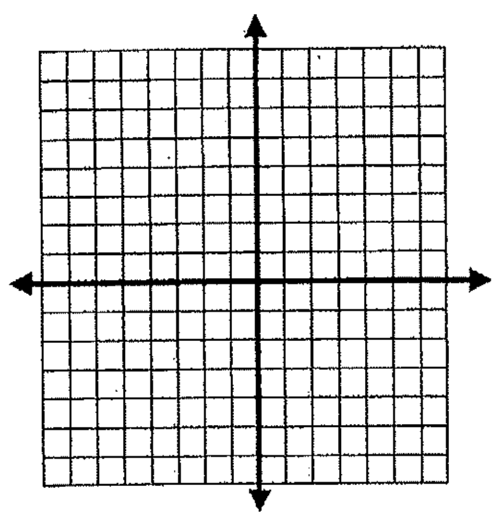
10. $f(x)=\frac{3x^2-6x-24}{5x^2-26x+5}$

Graph each absolute value function. Then write the absolute value function as a piecewise function that is formed by two lines of the form $y=mx+b$.

11. $f(x)=|2x+3|$



12. $f(x)=\left|\frac{1}{3}x-4\right|$



Simplify.

13. $\frac{4x^3-2x^2+3}{\sqrt{x}}$

14. $\frac{3x^{2/3}+5x-9}{\sqrt[3]{x}}$

15. $4^{3/2}$

16. $27^{4/3}$

Solve each equation for x.

17. $ax + b = 3(x - a)$

18. $-x = (5x + 3)(3x + 1)$

19. $\frac{2a}{x-1} = a - b$

Use synthetic division.

20. $x^3 - 4x^2 - 5$ is divided by $x - 3$

21. $-3x^4 - 2x - 1$ is divided by $x - 1$

22. $4x^3 - 3x^2 - 8x + 4$ is divided by $x - 2$

Simplify.

23. $\frac{\sqrt{x-5}}{3} - \frac{2}{\sqrt{x-5}}$

24. $(2x - 1)^{1/2} + 3(2x - 1)^{-1/2}$

25. The perimeter of a triangular plot of land is 2400 feet. The longest side is 200 ft less than twice the shortest. The middle side is 200 ft less than the longest side. Find the lengths of the three sides of the triangular plot.

26. Let x represent one of two positive numbers whose sum is 30.

a) Represent the other number in terms of x.

b) What are the restrictions on x?

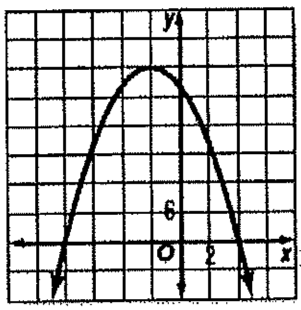
c) Write the function $f(x)$ the represents the product of these two numbers.

d) Find the two numbers if their product equals 104.

38. Use the table below to answer the following questions:

a) $f(g(1))$ b) $g(f(-1))$ c) $f(f(-1))$ d) $g(f(0))$ e) $f(g(-2))$

x	-3	-2	-1	0	1	2	3
f(x)	-7	-5	-3	-1	3	5	5
g(x)	8	3	0	-1	0	3	8

39. 

Identify the values of x at which $f(x) = 0$.

Over which interval(s) is $f(x) < 0$?

Over which intervals is $f(x)$ decreasing?

40. Simplify.

$$f(x) = 2x^2 + 3x - 1, \quad \frac{f(x+h) - f(x)}{h}$$

41. $\left(\frac{2}{7} - \frac{5}{14} + \frac{1}{8}\right) - \left(\frac{5}{7} + \frac{1}{4} - \frac{9}{2}\right) =$

42. Solve for y'. $3xy + 7y^3y' - 12x^2y - 21y' = 0$.

43. Write an equation of the line that passes through $(2, f(2))$ and $(-3, f(-3))$.

$f(x) = 2x^2 - 7x + 1$

44. Solve the equation for x. $(3-x)^{-\frac{1}{2}} + \frac{\sqrt{3-x}}{4} = 1$

Find the inverse of each function.

27. $y = 3x - 4$

28. $y = \frac{1}{x-3}$

29. If $f(3) = -2$, what is the value of $f^{-1}(-2)$?

Condense each logarithm.

30. $2\log_3(x-4) + \frac{1}{2}\log_3 x - \log_3(x+2)$

31. $\log_a m + 2\log_a n + 3\log_a p - \log_a n$

Expand each logarithm.

32. $\log_4 \sqrt[3]{\frac{x^5y^2}{w^6}}$

33. $\log_a \frac{x^2\sqrt[3]{x-3}}{(2x-1)}$

34. Write the equation of the line that passes through the point $(-9, 5)$ and has a slope of -2. Write this equation in point-slope form.

35. An object is propelled vertically upward with an initial velocity of 20 meters/second. The distance s (in meters) of the object from the ground after t seconds is $s = -4.9t^2 + 20t$. **(You may use a calculator for this problem.)**

a) When will the object be 15 meters above the ground?

b) When will the object strike the ground?

c) What is the maximum height? (hint: vertex)

36. The circumference of a circle is directly proportional (or varies directly) to the radius. A circle with a radius of 7 cm has a circumference of 43.96 cm. Find the circumference of the circle if the radius changes to 11 cm. **(You may use a calculator for this problem.)**

37. If the half-life of a radioactive substance is 3 years, and there is initially 600 grams, how much of the substance will be present after 6 years?

45. $\sin\left(\frac{\pi}{6}\right)$ ______

46. $\cos\frac{2\pi}{3}$ ______

47. $\tan\left(\frac{\pi}{4}\right)$ ______

48. $\sin\left(-\frac{\pi}{6}\right)$ ______

49. $\tan\pi$ ______

50. $\csc\frac{5\pi}{6}$ ______

51. $\cos\left(\frac{\pi}{2}\right)$ ______

52. $\cos\frac{3\pi}{4}$ ______

53. $\tan\frac{\pi}{6}$ ______

54. $\cos^{-1}\left(\frac{1}{2}\right)$ ______

55. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ ______

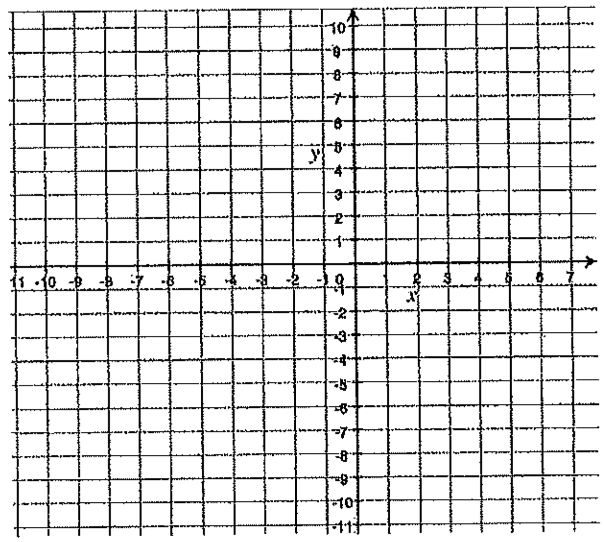
56. $\tan^{-1}(1)$ ______

57. $f(x) = \begin{cases} |x+7|-2 & for \quad -10 \le x < -3 \\ 2 & for \quad -3 \le x < 1 \\ -3x+8 & for \quad x \ge 1 \end{cases}$

Is this function continuous at $x = -3$?

Is this function continuous at $x = 1$?

Graph the function.



For problems **#58-59**, use the function $f(x) = -2x^2 - 5x + 3$.

58. Find $\frac{f(x+h)-f(x)}{h}$.

59. Find the average rate of change from $x = 2$ to $x = 5$.

Sketch a graph of each function and state its domain.

60. $f(x) = \sqrt{x-4}$

61. $f(x) = -x^2 - 9$

62. $f(x) = x^3 - 8$

63. $f(x) = \ln x$

64. $f(x) = e^x + 2$

65. $x^2 + y^2 = 16$

66. $f(x) = |x^2 - 2x - 15|$

67. $f(x) = \frac{1}{x-2}$

68. $f(x) = [\![x]\!]$

69. $f(y) = y^2$

70. $x = 3y - 1$

71. $y = \sin x$

72. $y = \frac{1}{|2x-1|}$

73. $y = \cos x$

74. $y = \tan x$

75. Use long division. $\frac{x^4-3x^3-4x^2+12x}{x^2+1}$

Use trigonometric identities to simplify.

76. $\frac{cos^2\theta - sin^2\theta}{sin\theta\ cos\theta}$

77. $sin^2\theta(csc^2\theta - 1)$

78. $\frac{1-sin^2\theta}{cos\theta}$

79. $\frac{\tan\theta}{\sec\theta}$

Solve each equation.

80. $\sin x = \sin 2x$

81. $\sin 3x = -1$

82. $\cos 2x = \frac{\sqrt{3}}{2}$

83. $x^2 - 2x - 8 < 0$

84. $\left|\frac{1}{3}x + 2\right| < 1$

85. $\frac{x-6}{x+2} > -1$